Flutter of a Finite Rectangular Wing'D Mill in Pitch and Heave

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Abridged: Introduction

As early aircraft increased in speed, novel self-amplifying coupled oscillations between control surfaces and bending wings or twisting rear fins caused many structural failures and crashes[1]. For instance the inertia of an unbalanced aileron acting at a cg behind its hinge makes it overshoot the wing at the top of a bending oscillation, so dropping the wing lift to amplify its rebound down. But the voluminous linear computational & experimental flutter airspeed literature did not even note that there can be two roots. Proving flutter could cease in high winds was the first step towards pursuing flutter windpower. (Sir James Lighthill FRS had stressed the need for storm protection) . In 1980 storm stability of- pitch and roll flutter had been hypothesized and proven successively with models on top of a car and then in Southampton, CEGB Marchwood and UCL windtunnels, by computations at Gifford & Partners and later in full size prototype #3 in gales and finally algebraically [2]. Very crudely the pitch swing to the inclination of the tailheavy wing is decreased by the rise in pitch damping as windspeed so the pitch amplitude decreases with windpseed, avoiding high wing loadings in the windmill low drag equivalent of the high thrust efficiency of oscillating propulsion in nature. Gust response is neutral at cutout and is more and more damped above, with the wing feathering to the mean wind at midspan whilst inclining away from the roll torque of any shift of the upper wind. (However a sweptback wing doesn't cutout but basically diverges in high winds).

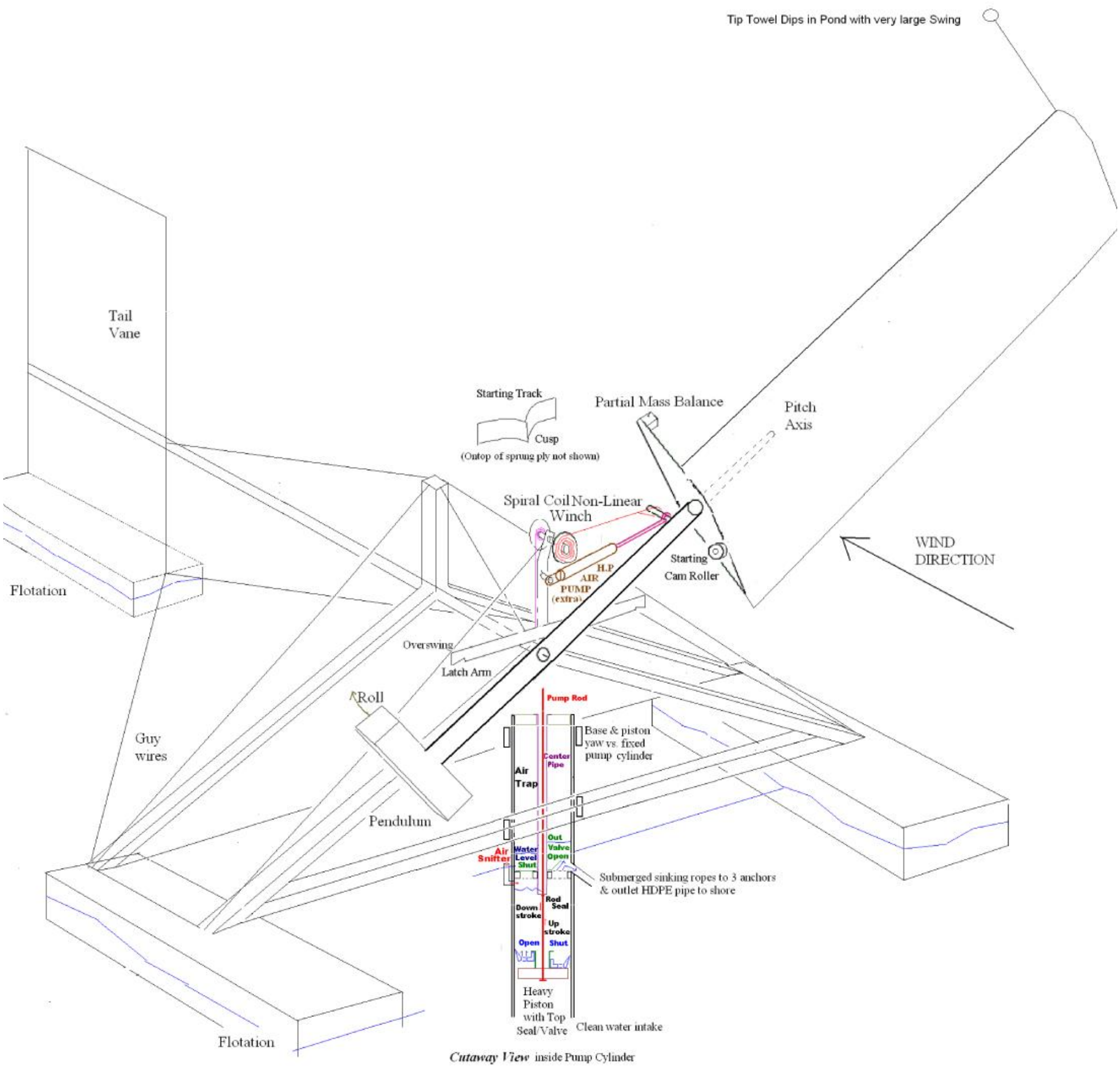


Fig 1 Perspective of Floating *Flo*'Pump with pump down and up stroke cutaways   ~~(cFlopump.tif)~~

Duncan [1] had pedantically demonstrated that the airstream powers flutter's unstable oscillations with his model "wing engine", a balanced wing articulated to pitch in set quadrature to heave (plunge). The BBC 1976 Young Scientists' more practical pitch and roll (Fig 1) free flutter model lacked a pump to prove better wind waterpumping than fanmill windpumps without the dangerous climbs. (Pocklington School attributed the idea to Sir. G.I. Taylor, but his biographer G.K Batchelor FRS could not find it in GIT's papers). In 1980 my sprung model with a cam track to depress a honed piston pumped to high psi. The power varies with amplitude squared in linear theory, but pitch saturates at ±90° and ultimately the power must vary as large sweeping roll amplitude. In 1993 full-scale smoke [3] found a beneficial extraordinary leading edge vortex shed at full ±90° pitch flip. Definitive confirmation of economic power far outweighing the fatigue penalties of 40% with steel and 20% with wood came with this fullscale prototype base #7 on a test well with instruments read by a computer in a trailer at the BC Science Council. Altogether 8 bases and 6 wings were prototyped. Scaling shows all inertially unbalanced fluid machines (incl. H Vawts and orthinopters) have an upper limit to design flowspeed as the square root of the strength to weight ie the endurance limit stress/ material density[2] and so the FWP niche is a light wind regime (and also gravity-dominated large size)

The rotary multiblade windpump loses all the kinetic energy of the swirl reaction to its high torque [4], which still can't turn over its crank to start in the best wind for its stroke. For a typical wind regime it can only usefully pump just 10% of the work of a Betz ideal windmill [5]. Duncan's fixed cycle wing engine would likewise stall against the fixed head of a piston pump. The free amplitude of flutter instability is inherently suited to reciprocating a pump whose stroke needs to vary to efficiently capture the changeable wind. The rectangular Flutterwing free to pitch (360°) on top of a roll pendulum winds a spiral winch pump (Figs 1 &2) with stroke varying as almost the cube of roll amplitude. So the pump stroke increases strongly with roll, not loading a starting swing in only 1m/s, but still absorbing up to ½ of the Betz wind energy swept in a big roll in a good wind. Compressing an aircylinder on the return stroke is very simple but not so supra-linear so a flow rate of about 3 cfm to a pressure of 4 atm for 250W was the max for a total 400 peak pumping W (eg 4L/s @10m head) in 3.5m/s by an 5.5 x1.37x.2 m 20 kg wing. Wind gradient and the non-linearities of the stiffening of the single-acting pumping, large amplitude pitch, and crosswind roll speed exceeding windspeed all increase the power[6] and the cutout causing a hysteresis vs. the linear restart wind. Key to curbing any overswing is that flutter pitch and roll are partly in phase so the Flo'Pump's tip balance rod makes first contact with the pond to wet its absorbent material and temporarily reduce the tailheaviness that flips the wing. A simple cusp track ensures self-starting from small amplitude in very light and steady winds but misses wing cam contact at bigger amplitude or when excess winds blow it back[7]. The *Flo*'Pump prototype Fig 1 floating around its pump cylinder with upper air chamber pumped through its underwater outlet pipe to shore and overland powerfully and reliably for 10 years with its catamaran righting moment limiting the stresses. Cantilevering can be eliminated by putting the pendulum inside the frame with the winch turned around and the pump and yaw axis better in front. It needs help with commercialisation or adoption in the Third World or making very large prototypes to assist pumped hydro.

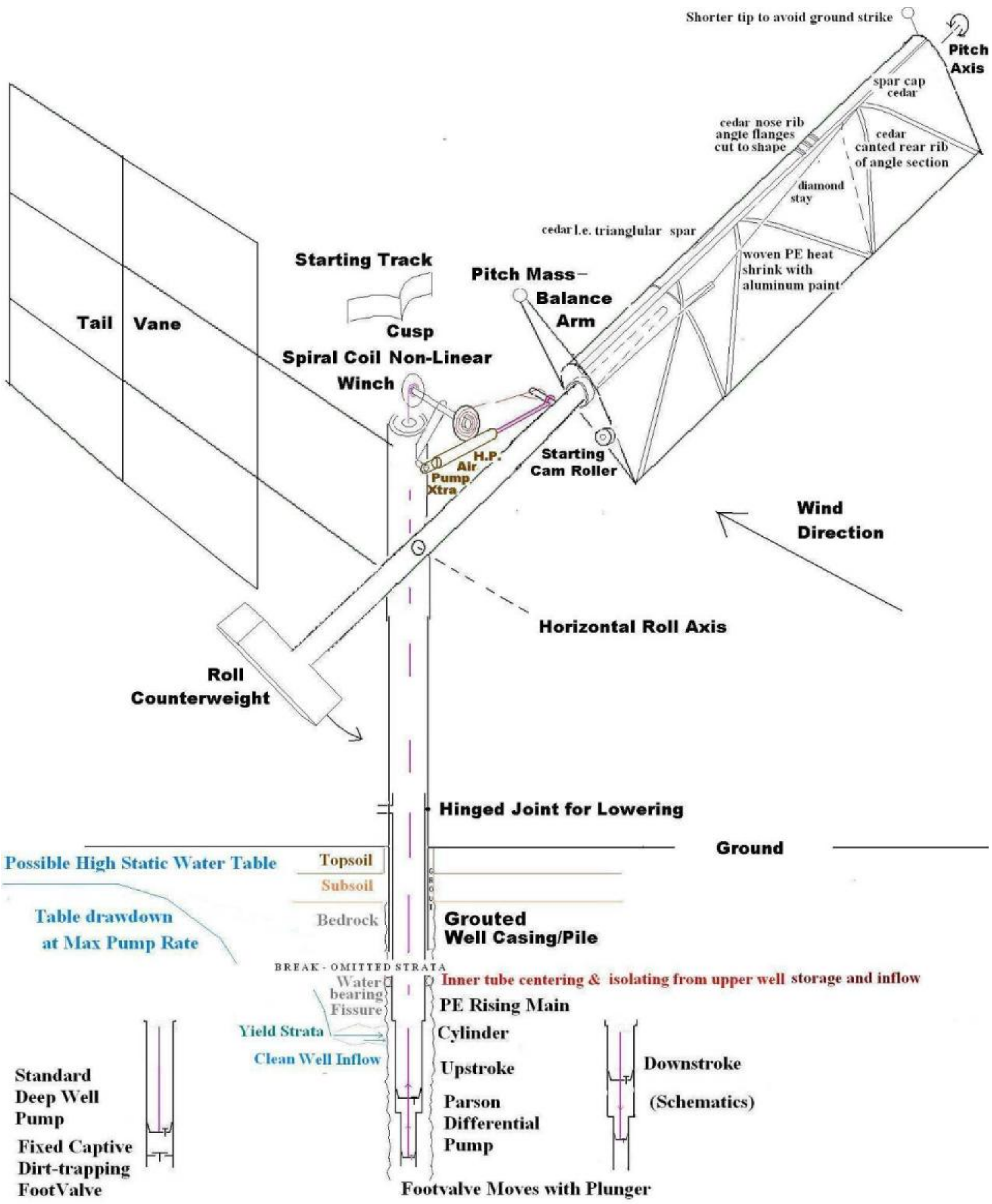
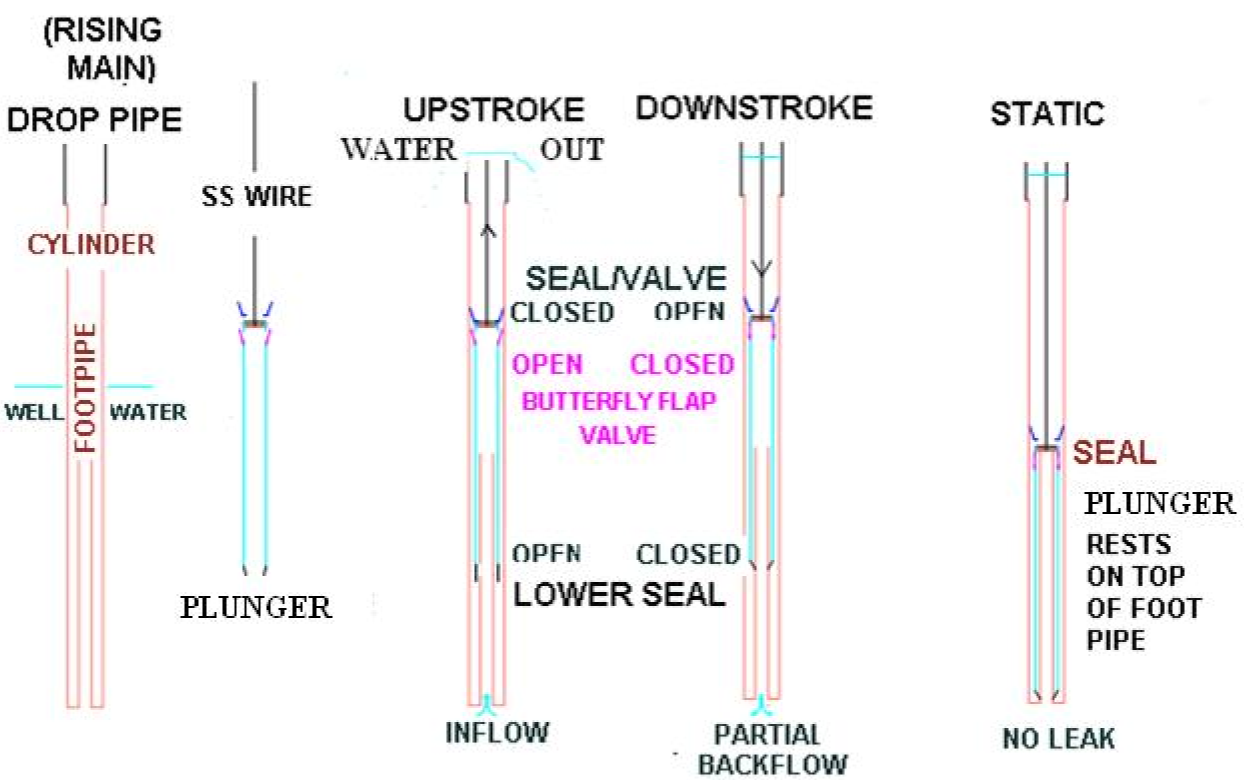


Fig 2 Schematic of the Flutterwell Pump, the Well-mounted FlutterWing Pump

The Flutter*well* base in Fig 2 uses the steel well casing as a short foundation pile. The shorter tip rod and counterweight is heavily sprung against impact and lightly sprung to slide forward to reduce the pitch cross inertial forcing by roll deceleration. Then after a long calm the Flutterwell could pump enough easy large, but safe, strokes at lowhead to drawdown the well and a high static watertable to a sustainable yield. The well storage drawdown transient can be minimised by inflating a ring seal between the rising main and the well just above the strata yielding water. Some of the wind's varying power is inevitably wasted in the varying head of groundwater resistance.

The novel differential area pump keeps the pumpwire in tension for downward accelerations up to its upper/lower area ratio>1 times Newton's *g*. Pulling its plunger with all the seals and valves out of the cylinder releases the water column for easy retrieval and maintenance. Its design for low internal flow constriction Fig 3 uses Fig 1's single part seal valve this time just above a perimeter rubber flap valve over a grid of holes, and a ring seal inside the bottom of the plunger. A snifter, NRV and airtank have been tested at the wellhead for overland pumping overcoming air absorption and indeed outputting some compressed air. A spinoff is a powerful well footpump[9]. But a fluttering watermill is elusive, only demonstrated to moderate amplitude via the dominance of static gravity imbalance in pitch and roll in water[10]

Fig 3 Detail of the actual differential well windmill and footpump



Despite being ubiquitous in early aircraft, flutter has scarcely been a problem in marine hydrofoils. The ratio of foil mass to the virtual mass ***m*** of the circumscribing fluid cylinder is <<1 with the 700x water density whereas aircraft wing mass ratios are >>1. Pitch and heave flutter calculations of typical hydrofoils by eminent aeroelasticians [11] backed up their empirical lower flutter limit of roughly unit ratio. An upside down version of Fig 1 with two opposed ferrocement blades could be not be adjusted to flutter when towed through water.

So the basic heaving fluttermill of chord ***c***, virtual mass/unit length ***m***, that is free to pitch ¥= about an axis ***ec*** ahead of the ¼ chord aerodynamic center has been analysed in a series of papers [2,12,13]. The lack of=¥ mechanical stiffness allowed a first ever algebraic solution of binary flutter drawing all the neutrally stable frequency μ contours in total mass imbalance ***xmc*** vs pitch inertia jmc^2 space. All contours pass through a nexus **N**=(mo^2,***mo***) of the same total inertia and imbalance as just ***m*** mislocated (too far aft) at the ¾ chord point ***o*** behind the pitch axis [12] where the nominal upwash $U_{3/4}$ and so the wake and circulation vanish for all μ. The apparent mass/span of the finite (thin) wing *is* ***lm***, $l \approx 1-½A^{-2}$. Then *a priori* for a 2D ¾ chord line offset ***o*** to the pitch axis the flow is pure potential implying neutrality of any flutter at **N** =(lmo^2, ***lmo***) because again $U_{3/4}$ =0 everywhere by the heave equation

Actual hydrofoil virtual and real (structural) ***j/x*** are closer to the pitch axis and so are on the left side of the 'ray' from (0,0) to **N**. Even an understressed solid steel hydrofoil has ***x*** short of the low frequency limit line 'lfl'. With pitch axis lead ***e***>0, this quasi-steady "lfl" line through **N** extends downwards in ***x*** with ***j*** to reduce the tailheaviness ***x*** for flutter (even to negative ***x*** or noseheaviness.) Generally the larger flutter zone of higher ***m***, ***j*** and ***j/x*** (partially mass-balanced wings) to the right of the nexal ray and above the lfl allows more heave/pitch for heavier-than-air Wing'd Pump power with high wind ***V*** cutout in pitch and roll.

The binary discriminant σ of this biquadratic in ***j*** and ***x*** is proportional to the discriminant δ of the quadratic in ***e*** for pure pitch flutter[13] :

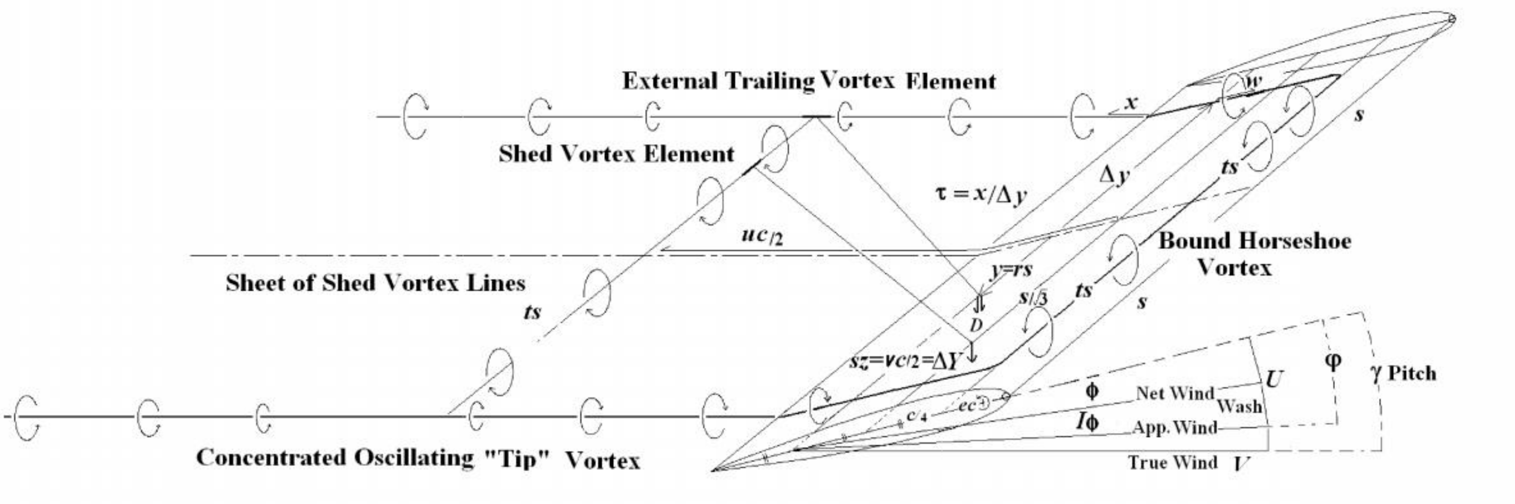
$$4F^2\sigma/k^4=\delta=(2g-F-½)^2-2F=\{2g-F-½-\sqrt{2F}\}\,[2g-F-½+\sqrt{2F}]=\{2g-(\sqrt{F}+1/\sqrt{2})^2\}[2g-(\sqrt{F}-1/\sqrt{2})^2] \qquad (1)$$

where ***T=F-iG=F-ikg*** Theodorersen's famous 1933 1D wake function of reduced frequency ***k***=μ***c/V***. Now as $k\uparrow\infty$, $F\downarrow½+¼/k^2$ and $G\downarrow¼/k$ so by extraordinary cancellation $\sqrt{2g}>\sqrt{F}-1/\sqrt{2}$ making the 1D high frequency contours ellipses, from **N** to 4**N**[13], kissing the lfl line at **N**). Any 2D change from this special 1D lower limit of ***F*** of ½ will open the high frequency contours into hyperbolae crossing the lfl line. (The back induced flow is $\int_0^\infty d\Gamma(x)/\,2\pi(x+c/2)$ or by parts -d(0)/π***c*** in this limit, the same as in a Wagner sudden angle of attack so the induced fraction ***I***=1 and the lift ½ the steady in both cases[13].) As $k\downarrow0$ a negative Ln singularity in the 1D ***g*** and so in the net pitch damping allows pure pitch flutter at $k_z\leq.087$ of {}≤0 for δ<0, (with very airfoil high pitch inertia and the wake vortices fed by the drag work.) This hyperbolically repels the $k\approx k_z$ binary contours, even bizarrely back to very large negative imbalance but no inertia at about $½k_z$. [13] Here 2D finite aspect ratio *A* is added for a vital reality check.

2. 2D Aerodynamic model

Jones also corrected its added mass $\boldsymbol{m}$ by $\boldsymbol{l}$ for aspect ratio. Assuming this henceforth everywhere, then the general 1D flutter solutions [13] are conveniently retainned if $L\approx 1+½A^{-2}$ multiplies $\boldsymbol{j,x}$ &$\boldsymbol{T}$.

The A=4 Flutterwings are statically stable in pitch at 23.5% chord ahead of the 23.7% cp from O(A^{-2}) lifting surface theory and experiment[17], weakly moved from the 1D ¼ chord, so that 1D cp will be retainned. The circulatory lift ®°$cVTU_{¾}$ where Theodorsen's T=1/ 1+I_s. $I_s=K_0(v)/K_1(v)$ with v=½ik and K modified Bessel functions of the second kind. I is the fractional loss (induced=apparent-net) of angle of attack (vs the net) due to the wake. (Fig 4), like the constant $I_q=N$-1 along an elliptical wing in steady lifting line theory long extended empirically for rectangular wings to O(A^{-2}) and further here to oscillating flow as I_t. Then the average trailing I_t and shed I_s downwashes can be summed for a combined loss, a logical symmetry missing in Jones.



From conclusions:

But pitch and roll are also coupled by gravity static imbalance which is dominant over the above dynamic imbalance at low stream speeds. A very long pitch counterbalance arm of 3.5c above water provided sufficient coupling and pitch inertia for a c=15cm foil to flutter in water[10]. At full scale[2] enough coupling might come from a 1c arm that could stay clear of the water until max roll. Perhaps, the low phase shift and amplitude ratio linear instability at low inertia to the left of the e=0 nexus would distort non-linearly to higher amplitude ratio and power. Otherwise above water a pitch gear up to a flywheel [12] could increase the pitch inertia above **N**'s ¼ lmc^2. A simple first test would be below a low steel bridge over a small slow river. Instead of the winch, an immersed rhombus could invert the increase in distance with swing into non-linear compression on its diagonal for pumping.

In all, this paper has given the state of the art of power flutter, from the proven Flo'Pump to the Flutterwell pump which needs testing on real wells to final proof of the impracticality of pure pitch flutter and finite aspect ratio extension of the search for hydrofoil variants.

Update:

No-load model releases of from 165° roll and no pitch do not reattain 165° for several cycles, but if roll 180° bdc is exceeded the model wing autorotates (in roll) with a high tip speed ratio centrifuging even its minimised planar pitch inertia to near tangential pitch. Whereas a fishing line limiting the excursion of the te from tdc [8] prolongs the nose down pitch in a roll swing past 110° and reduces the wing flip so the next opposite roll is safely smaller, but this would require bumper stops to prevent complete pitch rotations and moving the starting cam as far back as possible, etc.

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